

$$\alpha = -\text{sign}(x_1) \|x\|_2$$

1. set Norm 2

$$H = I - \beta v v^T$$

2. set  $\alpha$

$$\beta = \frac{2}{v^T v}$$

3. find  $v$

4. set  $\beta$

5. find  $H$

## Lecture 10 : Linear Least Square

วิธีแก้ solve linear least square  
ใช้ QR factorization หรือ Householder

QR factorization (4-5)  
Householder (1-3)  
Givens Rotation

Use Householder QR factorization to solve:

$$\begin{bmatrix} 1 & 5 & 3 \\ 2 & -1 & 3 \\ -2 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}_{4 \times 3} x \cong \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

1. Annihilate first column

$$a = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 2 \end{bmatrix}$$

$$\|a\|_2 = \sqrt{1^2 + 2^2 + (-2)^2 + 2^2} = \sqrt{13}$$

$$\alpha = -(\cdot)(\sqrt{13}) = -\sqrt{13}$$

$$v = a - \alpha e_1$$

$$= \begin{bmatrix} 1 \\ 2 \\ -2 \\ 2 \end{bmatrix} - (-\sqrt{13}) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ -2 \\ 2 \end{bmatrix} + \begin{bmatrix} \sqrt{13} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 4.605 \\ 2 \\ -2 \\ 2 \end{bmatrix}$$

$$\beta_1 = \frac{2}{v_1^T v_1}$$

$$= \frac{2}{\begin{bmatrix} 4.605 & 2 & -2 & 2 \end{bmatrix} \begin{bmatrix} 4.605 \\ 2 \\ -2 \\ 2 \end{bmatrix}}$$

กำลัง 2 ของแต่ละตัว  
บวกกัน

$$\beta_1 = 0.0602$$

$$H_1 = I - \beta_1 v_1 v_1^T$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - 0.0602 \begin{bmatrix} 4.605 \\ 2 \\ -2 \\ 2 \end{bmatrix} \begin{bmatrix} 4.605 & 2 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - 0.0602 \begin{bmatrix} 21.22 & 9.21 & -9.21 & 9.21 \\ 9.21 & 4 & -4 & 4 \\ -9.21 & -4 & 4 & -4 \\ 9.21 & 4 & -4 & 4 \end{bmatrix}$$

$$H_1 \approx \begin{bmatrix} -0.2777 & -0.555 & 0.555 & -0.555 \\ -0.555 & 0.759 & 0.241 & -0.241 \\ 0.555 & 0.241 & 0.759 & 0.241 \\ -0.555 & -0.241 & 0.241 & 0.759 \end{bmatrix}$$

$$H_1 A = \begin{bmatrix} -3.6056 & 0.2774 & -3.6056 \\ 0 & -3.0509 & 0.1315 \\ 0 & 4.0509 & 3.8685 \\ 0 & -2.0509 & 0.1315 \end{bmatrix}$$

2. Annihilate second column of submatrix

$$H_1 A = \begin{bmatrix} -3.6056 & 0.2774 & -3.6056 \\ 0 & -3.0509 & 0.1315 \\ 0 & 4.0509 & 3.8685 \\ 0 & -2.0509 & 0.1315 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -3.0509 & 0.1315 \\ 4.0509 & 3.8685 \\ -2.0509 & 0.1315 \end{bmatrix}$$

$$a = \begin{bmatrix} -3.0509 \\ 4.0509 \\ -2.0509 \end{bmatrix}$$

$$\|a\|_2 = 5.47103$$

$$\alpha = -(\cdot)(5.47103) = -5.47103$$

$$v_2 = a - \alpha e_1$$

$$= \begin{bmatrix} -3.0509 \\ 4.0509 \\ -2.0509 \end{bmatrix} - (-5.47103) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -8.5212 \\ 4.0509 \\ -2.0509 \end{bmatrix}$$

$$\beta_2 = \frac{2}{v_2^T v_2}$$

$$= \frac{2}{(-8.5212)^2 + (4.0509)^2 + (-2.0509)^2}$$

$$= 0.02145$$

$$H_2' = I - \beta_2 v_2 v_2^T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 0.02145 \begin{pmatrix} \begin{bmatrix} -8.5212 \\ 4.0509 \\ -2.0509 \end{bmatrix} \begin{bmatrix} -8.5212 & 4.0509 & -2.0509 \end{bmatrix} \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 0.02145 \begin{bmatrix} 72.6108 & -34.5185 & 17.4761 \\ -34.5185 & 16.4097 & -8.3079 \\ 17.4761 & -8.3079 & 4.2061 \end{bmatrix}$$

$$H_2' \approx \begin{bmatrix} -0.5575 & 0.7404 & -0.3748 \\ 0.7404 & 0.6480 & 0.1782 \\ -0.3748 & 0.1782 & 0.9097 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & H_2' & \\ 0 & & & \end{bmatrix}$$

$$H_2 H_1 A = \begin{bmatrix} -3.6056 & 0.2774 & -3.6056 \\ 0 & 5.4701 & 2.7421 \\ 0 & 0 & 2.6274 \\ 0 & 0 & 0.7598 \end{bmatrix}$$

3. Annihilate third column of submatrix

$$H_2 H_1 A = \begin{bmatrix} -3.6056 & 0.2774 & -3.6056 \\ 0 & 5.4701 & 2.7421 \\ 0 & 0 & 2.6274 \\ 0 & 0 & 0.7598 \end{bmatrix} \rightarrow A_3 = \begin{bmatrix} 2.6274 \\ 0.7598 \end{bmatrix}$$

$$a_3 = \begin{bmatrix} 2.6274 \\ 0.7598 \end{bmatrix}$$

$$\|a_3\|_2 = 2.735$$

$$\alpha = -(+)(2.735) = -2.735$$

$$v_3 = a_3 - \alpha e_1$$

$$= \begin{bmatrix} 2.6274 \\ 0.7598 \end{bmatrix} + 2.735 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5.3624 \\ 0.7598 \end{bmatrix}$$

$$\begin{aligned} \beta_3 &= \frac{2}{v_3^T v_3} \\ &= \frac{2}{(5.3624)^2 + (0.7598)^2} \\ &= 0.0681 \end{aligned}$$

$$H_3' = I - \beta_3 v_3 v_3^T$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 0.0681 \begin{pmatrix} \begin{bmatrix} 5.3624 \\ 0.7598 \end{bmatrix} \begin{bmatrix} 5.3624 & 0.7598 \end{bmatrix} \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 0.0681 \begin{bmatrix} 28.7553 & 4.0743 \\ 4.0743 & 0.5772 \end{bmatrix}$$

$$H_3' = \begin{bmatrix} -0.9606 & -0.2778 \\ -0.2778 & 0.9606 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & & \\ 0 & & H_3' \end{bmatrix}$$

$$H_3 H_2 H_1 A = \begin{bmatrix} -3.6056 & 0.2774 & -3.6056 \\ 0 & 5.4701 & 2.7421 \\ 0 & 0 & -2.7350 \\ 0 & 0 & 0 \end{bmatrix} = R \quad \text{and} \quad Q = H_1 H_2 H_3$$

can check  $Q^T A \approx R$  and  $Q^T Q = I$

এভাবে প্রদত্ত QR Factorization (Solving Least-Square like) অর্থোমিটারিক্যাল Q and R বোঝায়।

4. Compute  $c = Q^T b$

$$= H_3 H_2 H_1 b$$

$$\therefore C = \begin{bmatrix} 0 \\ 2.1937 \\ -0.36 \\ -0.2410 \end{bmatrix}$$

5. Solve  $R_1 x = c_1$  for  $x$

$$C = \begin{bmatrix} 0 \\ 2.1937 \\ -0.36 \\ -0.2410 \end{bmatrix}$$

(compatibility)

$$A = \begin{bmatrix} 1 & 5 & 3 \\ 2 & -1 & 3 \\ -2 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

A is  $4 \times 3$ , only first 3 rows of R carry equation

$4 \times 3$

$$\therefore C_1 = \begin{bmatrix} 0 \\ 2.1937 \\ -0.36 \end{bmatrix}$$

$$\therefore R_1 = \begin{bmatrix} -3.6056 & 0.2774 & -3.6056 \\ 0 & 5.4701 & 2.7421 \\ 0 & 0 & -2.7350 \end{bmatrix}$$

$$R_1 x = c_1$$

$$\begin{bmatrix} -3.6056 & 0.2774 & -3.6056 \\ 0 & 5.4701 & 2.7421 \\ 0 & 0 & -2.7350 \end{bmatrix} x = \begin{bmatrix} 0 \\ 2.1937 \\ -0.36 \end{bmatrix}$$

$$x = \begin{bmatrix} -0.1058 \\ 0.3351 \\ 0.1316 \end{bmatrix} \quad \#$$