

Always compute midpoint: $m = a + \frac{b-a}{2}$

Lecture 11: Nonlinear Equations
Interval Bisection Method
(Non-linear equation in 1D)

ถ้า $f(m)$ Sign ตรงกับ $f(a)$ แล้ว $\rightarrow$ update b
ถ้า $f(m)$ Sign ตรงกับ $f(b)$ แล้ว $\rightarrow$ update a } ใช้จนกว่าจะหาค่าที่ใกล้เคียง
อันไหนมาขึ้นกับ

Perform the **interval bisection** to find an approximation to a root of $f(x) = x^2 - 3x - 4 = 0$

Setup: Given interval $[a, b]$ an interval where we require $f(a)$ and $f(b)$ have opposite sign

a $f(3) = 3^2 - 3(3) - 4 = -4$ (-)
b $f(6) = 6^2 - 3(6) - 4 = 14$ (+)

OK!

$\therefore$ Start with interval $[3, 6]$

Iteration 1:

Initial. $a = 3, b = 6$

Midpoint: $m = 3 + \frac{6-3}{2} = 4.5$

$f(4.5) = (4.5)^2 - 3(4.5) - 4$
 $= 2.75$ (different sign $f(a) \rightarrow$ update b)

$\therefore$ New interval: $[3, 4.5]$

Iteration 2:

Current. $a = 3, b = 4.5$

Midpoint: $m = 3 + \frac{4.5-3}{2} = 3.75$

$f(3.75) = (3.75)^2 - 3(3.75) - 4$
 $= -1.1875$ (different sign $f(b) \rightarrow$ update a)

$\therefore$ New interval: $[3.75, 4.5]$

Iteration 3:

Current. $a = 3.75, b = 4.5$

Midpoint: $m = 3.75 + \frac{4.5-3.75}{2} = 4.125$

$f(4.125) = (4.125)^2 - 3(4.125) - 4$
 $= 0.640625$ (different sign $f(a) \rightarrow$ update b)

$\therefore$ New interval: $[3.75, 4.125]$

: ~~more iterations~~

but if we stop here

$\therefore$ Final approximation: $3.75 + \frac{4.125-3.75}{2} = 3.9375$ # (True root: $x^* = 4$)

$\therefore$ Maximum error: $\frac{4.125-3.75}{2} = 0.1875$