

Lecture 15: Eigenvalues and Eigenvectors

Normalized power method

Wikipedia overflows power method

$$1. \tilde{x}^{(k)} = Ax^{(k)}$$

$$2. f^{(k)} = \|\tilde{x}^{(k)}\|_2$$

$$3. x^{(k+1)} = \frac{\tilde{x}^{(k)}}{f^{(k)}}$$

Perform **normalized power method** on $\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$

Iteration 0

$$1. \tilde{x}^{(k)} = Ax^{(k)}$$

$$\begin{aligned} \tilde{x}^{(0)} &= Ax^{(0)} \\ &= \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \tilde{x}^{(0)} &= \begin{bmatrix} 3 \\ -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} 2. f^{(k)} &= \|\tilde{x}^{(k)}\|_2 \\ f^{(0)} &= \sqrt{3^2 + (-1)^2} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} 3. x^{(k+1)} &= \frac{\tilde{x}^{(k)}}{f^{(k)}} \\ x^{(1)} &= \frac{\begin{bmatrix} 3 \\ -1 \end{bmatrix}}{\sqrt{10}} \end{aligned}$$

$$x^{(1)} = \begin{bmatrix} 3/\sqrt{10} \\ -1/\sqrt{10} \end{bmatrix}$$

Iteration 1

$$1. \tilde{x}^{(k)} = Ax^{(k)}$$

$$\begin{aligned} \tilde{x}^{(1)} &= Ax^{(1)} \\ &= \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 3/\sqrt{10} \\ -1/\sqrt{10} \end{bmatrix} \\ \tilde{x}^{(1)} &= \begin{bmatrix} 3.1622 \\ -1.897 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} 2. f^{(k)} &= \|\tilde{x}^{(k)}\|_2 \\ f^{(1)} &= 3.6878 \end{aligned}$$

$$\begin{aligned} 3. x^{(k+1)} &= \frac{\tilde{x}^{(k)}}{f^{(k)}} \\ x^{(2)} &= \frac{\begin{bmatrix} 3.1622 \\ -1.897 \end{bmatrix}}{3.6878} \end{aligned}$$

$$x^{(2)} = \begin{bmatrix} 0.8575 \\ -0.5145 \end{bmatrix}$$

$$\dots \quad x^{(7)} = \begin{bmatrix} 0.7126 \\ -0.7016 \end{bmatrix} \quad (\text{Not overflow now?})$$