

Lecture 15: Eigenvalues and Eigenvectors
QR Iteration Algorithm
ex7.py

1. $M^{(0)} = A$
 2. Factor $M^{(k)} = Q^{(k)} R^{(k)}$
 3. $M^{(k+1)} = R^{(k)} Q^{(k)}$
- } loop until desired M

Perform two steps of basic QR iteration on $A = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$

Iteration 0

$$M^{(0)} = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$M^{(0)} = \underbrace{\begin{bmatrix} -0.9701 & 0.2425 \\ 0.2425 & 0.9701 \end{bmatrix}}_{Q^{(0)}} \underbrace{\begin{bmatrix} -4.1231 & 2.4254 \\ 0 & 5.5783 \end{bmatrix}}_{R^{(0)}}$$

$$M^{(1)} = \underbrace{\begin{bmatrix} -4.1231 & 2.4254 \\ 0 & 5.5783 \end{bmatrix}}_{R^{(0)}} \underbrace{\begin{bmatrix} -0.9701 & 0.2425 \\ 0.2425 & 0.9701 \end{bmatrix}}_{Q^{(0)}} \quad (\text{don't mind the calculator garbage})$$

$$\therefore M^{(1)} = \begin{bmatrix} 4.5882 & 1.3529 \\ 1.3529 & 5.4118 \end{bmatrix}$$

Iteration 1

$$M^{(1)} = \begin{bmatrix} 4.5882 & 1.3529 \\ 1.3529 & 5.4118 \end{bmatrix}$$

$$M^{(1)} = \underbrace{\begin{bmatrix} -0.9592 & -0.2828 \\ -0.2828 & 0.9592 \end{bmatrix}}_{Q^{(1)}} \underbrace{\begin{bmatrix} -4.7836 & -2.8283 \\ 0 & 4.8081 \end{bmatrix}}_{R^{(1)}}$$

$$M^{(2)} = \underbrace{\begin{bmatrix} -4.7836 & -2.8283 \\ 0 & 4.8081 \end{bmatrix}}_{R^{(1)}} \underbrace{\begin{bmatrix} -0.9592 & -0.2828 \\ -0.2828 & 0.9592 \end{bmatrix}}_{Q^{(1)}}$$

$$\therefore M^{(2)} = \begin{bmatrix} 5.3882 & -1.3599 \\ -1.3599 & 4.6118 \end{bmatrix}$$

Off-diagonal elements are getting smaller!
The matrix gradually become diagonal



Diagonal entries are the eigenvalues of A