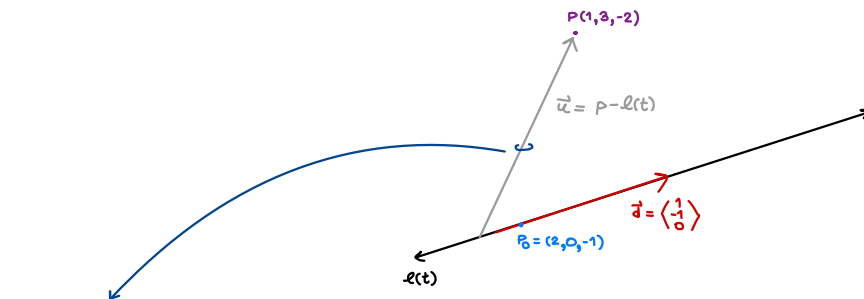


$$\ell(t) = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} t$$

Find the shortest distance from the point $p = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ to the parametric line

$\ell(t)$ through $p_0 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ with the direction vector $d = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$. Find the point p_1 which lies on the line and is closest to p



$$\vec{u} = p - \ell(t)$$

$$= \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} - \left(\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} t \right)$$

$$= \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} t$$

$$= \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} t$$

အကွာအဝေး

Distance between a point and a line

$$(\vec{u}, \vec{d}) = 0 \rightarrow \vec{u} \perp \vec{d}$$

$$\left(\begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} t \right) \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} t = 0$$

$$(1+3) + (1+1)t = 0$$

$$4+2t = 0$$

$$t = -2$$

$$= \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} (-2)$$

$$= \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 5 \\ -1 \end{bmatrix}$$

$$= \sqrt{35} \#$$

when t အကွာအဝေး 2 သက်သေ → intersection point

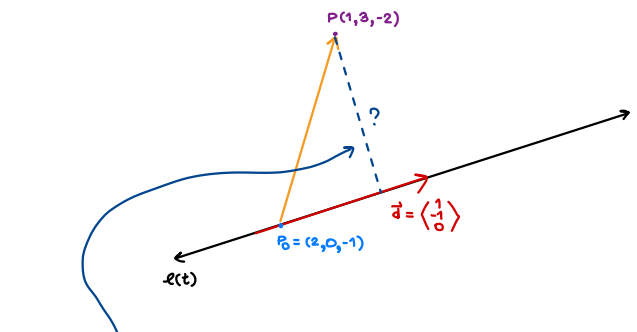
$$\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} (-2)$$

$$= \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \#$$

(វិធីសាស្ត្រ មេត្រី projection)

Find the shortest distance from the point $p = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ to the parametric line

$l(t)$ through $p_0 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ with the direction vector $d = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$. Find the point p_1 which lies on the line and is closest to p



$\vec{P} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$
 $= \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$

$\text{Proj}_d \vec{P} + ? = \vec{P}$
 $? = \vec{P} - \text{Proj}_d \vec{P}$
 $= \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} - \frac{(\vec{P} \cdot \vec{d})}{\|\vec{d}\|^2} \cdot \vec{d}$
 $= \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} - \frac{(\vec{P} \cdot \vec{d})}{\|\vec{d}\|^2} \cdot \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$
 $= \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} - \left(\begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right) \cdot \frac{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}{2}$
 $? = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} - \frac{-2}{2} \cdot \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} - \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$
 $|?| = \sqrt{3} \#$

an intersection point
 $l_1(t) = \begin{pmatrix} 2+t \\ -t \\ -1 \end{pmatrix}$
 $l_2(s) = \begin{pmatrix} 1+s \\ 3+s \\ -2-s \end{pmatrix}$
 $-2-s = -1$
 $s = -1$
 $\therefore$ Intersection point
 $(0, 2, -1) \#$